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BEHAVIOR OF THE ADAPTIVE LINE ENHANCER DURING INITIAL ADAPTATION

BA Bologna

11 October 1977

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ADMINISTRATIVE STATEMENT

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The behavior of the Adaptive Line Enhancer (ALE) during initial adaptation is analyzed analytically via the direct solution of the Widrow-Hoff least-mean-square (LMS) algorithm. The result shows that the initial transient response of the weight vector is approximately equal to $2\mu/T$ times the cross correlation vector of the input, i.e.,

mu

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20. Continued

2mu/T

Integral from tau = delta + ℓT to tau = t of

$$W_\ell(t) = \left(\frac{2\mu}{T}\right) \int_{\delta+\ell T}^{t} \left(x(\tau)\, x(\tau-\delta-\ell T)\, d\tau \right),$$

sub ℓ

tau delta

where μ is the adaptation constant, δ is the bulk delay of the filter, ℓ is the weight index and T is the sampling period or an element delay.

mu

delta

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INTRODUCTION

The adaptive LMS linear transversal filter has been used successfully in measuring narrowband spectral lines in broadband noise for cases of low signal-to-noise ratios. In this context it has been referred to as the Adaptive Line Enhancer (ALE) since this device alters its weight to enhance signals buried in noise.[1,2,3] In other applications it has been utilized successfully as a spectrum estimator.[4]

The weights of the ALE adapt in a manner such that the mean squared error is minimized. In all these applications its performance is usually analyzed in terms of the Wiener filter theory. To assess the detection performance and tracking capabilities of the ALE, the estimation of convergence time becomes important. The area pertaining to the overall convergence properties of the ALE has been extensively treated by John Treichler.[5] His thesis encompasses the mean steady state and mean transient behavior of the ALE in both stationary and non-stationary signal environments. The transient results show that the mean weight vector possesses an initial ramp response whose slope is related to the adaptive constant μ, the modal matrix $\bar{Q}$, and the cross correlation vector $\bar{P}_s$. Because the analysis is treated in a statistical mean, the fine structure of the weight vector, an oscillatory start-up phase, does not appear in the results.

This paper attempts to explain the mechanism associated with the initial transient response via direct solution of the LMS algorithm.

TRANSIENT BEHAVIOR

Conceptually the ALE is as shown in Figure 1. The LMS algorithm, i.e., the law which governs the time history of the weight vectors is given by:

$$W_s(n+1) - W_s(n) = 2\mu \left[x(n) - \sum_{\ell=0}^{L-1} W_\ell(n) \times (n - \ell - \delta) \right] \times (n - s - \delta).$$

Dividing both sides of the equation by a time increment Δ one obtains:

$$\frac{W_s(n+1) - W_s(n)}{\Delta} = \frac{2\mu}{\Delta} \left[\times (n) - \sum_{\ell=0}^{L-1} W_\ell(n) \times (n - \ell - \delta) \right] \times (n - s - \delta)$$

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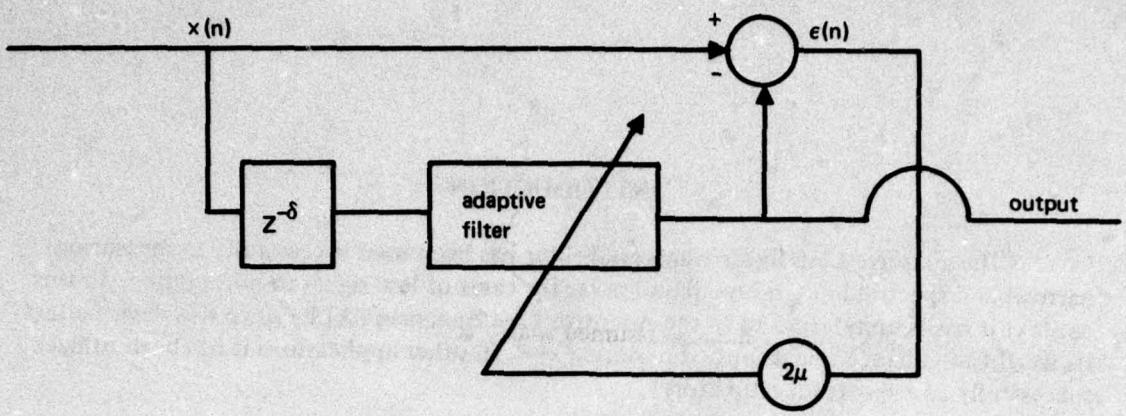

Figure 1. ALE configuration.

Define the L × 1 vectors as:

$$\underset{\circ}{\underline{W}}(n) = \frac{1}{\Delta}\begin{bmatrix} W_0(n+1) - W_0(n) \\ W_1(n+1) - W_1(n) \\ \vdots \\ W_{L-1}(n+1) - W_{L-1}(n) \end{bmatrix}, \quad \underline{P}(n) = \frac{2\mu}{\Delta}\begin{bmatrix} x(n) \cdot x(n-\delta) \\ x(n) \cdot x(n-1-\delta) \\ \vdots \\ x(n) \cdot x(n-L+1-\delta) \end{bmatrix},$$

$$\underline{W}(n) = \begin{bmatrix} W_0(n) \\ W_1(n) \\ \vdots \\ W_{L-1}(n) \end{bmatrix},$$

and the L × L matrix as:

$$A(n) = \frac{2\mu}{\Delta}\begin{bmatrix} x(n-\delta)\,x(n-\delta) & x(n-\delta)\,x(n-1-\delta) & x(n-\delta)\,x(n-2-\delta) \ldots .x(n-\delta)\,x(n-L+1-\delta) \\ x(n-1-\delta)\,x(n-\delta) & x(n-1-\delta)\,x(n-1-\delta) & x(n-1-\delta)\,x(n-2-\delta) \ldots .x(n-1-\delta)\,x(n-L+1-\delta) \\ x(n-2-\delta)\,x(n-\delta) & x(n-2-\delta)\,x(n-1-\delta) & x(n-2-\delta)\,x(n-2-\delta) \\ & & \\ x(n-L+1-\delta)\,x(n-\delta) & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & .x(n-L+1-\delta)\,x(n-L+1-\delta) \end{bmatrix}.$$

Then this notation simplifies to the difference equation

$$\underset{\circ}{\underline{W}}(n) = -A(n)\,\underline{W}(n) + \underline{P}(n),$$

whose continuous solution is

$$e^{\int A(t)\,dt}\,\underline{W(t)} = \int e^{\int A(t)\,dt}\,\underline{P(t)}\,dt + \underline{C},$$

and $\underline{C}$ is a constant vector of integration. Or

$$\underline{W(t)} = e^{-\int A(t)\,dt}\left[\int e^{\int A(t)\,dt}\,\underline{P(t)}\,dt + \underline{C}\right].$$

For all practical purposes, it shall be assumed that $\underline{C}$ is the null vector since this implies that all the initial weights are set to zero. Thus

$$\underline{W(t)} = e^{-\int A(t)\,dt}\int e^{\int A(t)\,dt}\,P(t)\,dt,$$

and

$$e^{-\int A(t)\,dt} = I - \int A(t)\,dt + \frac{1}{2!}\left[\int A(t)\,dt\right]\left[\int A(t)\,dt\right] \ldots \text{etc.}$$

Note that the matrix A(t) contains a multiplicative factor of 2μ, where $\mu \ll 1$. Thus for the initial transient time, the following approximation holds:

$$e^{-\int A(t)\,dt} \approx I - \int A(t)\,dt.$$

This leads to the following result:

$$\underline{W(t)} \approx [I - \int A(t)\,dt] \int [I + \int A(t)\,dt]\,\underline{P(t)}\,dt.$$

Further simplification is obtained by neglecting orders of 2μ greater than one:

$$\underline{W(t)} \approx \int \underline{P(t)}\,dt.$$

This result shows that the initial transient response of the weight vector is directly related to the cross correlation vector.

The analysis that follows will assume all terms whose order of μ is greater than one to be negligible. This assumption is valid for the initial adaptation since for sinusoidal inputs in white noise, the average weight vector varies as μt, so initial transient time will refer to all $\mu t \leq .1$ or $t \leq .1/\mu$.

APPLICATION

Consider the case where the input x(t) consists of a line contaminated by noise. Let

$$x(t) = A \text{ Sin } (w_0 t + \theta) + n(t),$$

where:

θ is a fixed phase

n(t) is zero mean, white noise.

Assume the filter to possess an initial bulk delay δ. The vector $\underline{P(t)}$ is then given by,

$$\underline{P(t)} = \frac{2\mu}{T}\begin{bmatrix} x(t)\,x(t-\delta) \\ x(t)\,x(t-\delta-T) \\ x(t)\,x(t-\delta-2T) \\ \vdots \\ x(t)\,x(t-\delta-(L-1)\,T) \end{bmatrix}$$

where T is an element delay of the ALE. Thus an element of the vector $\underline{W(t)}$ say $W_\ell(t)$ is given by

$$W_\ell(t) = \frac{2\mu}{T}\int_{\delta+\ell T}^{t} x(\tau)\cdot x(\tau-\delta-\ell T)\,d\tau.$$

For sinusoidal inputs,

$$W_\ell(t) = \frac{2A^2\mu}{\Delta}\int_{\alpha}^{t} \mathrm{Sin}\,(\omega_0\tau+\theta)\,\mathrm{Sin}\,(\omega_0(\tau-\alpha)+\theta)\,d\tau$$

$$\alpha = \delta + \ell T.$$

After performing the integration and evaluating the result at the limits, the following is obtained:

$$W_\ell(t) = \frac{A^2\mu}{\Delta}\left\{(t-\alpha)\,\mathrm{Cos}\,(\alpha\omega_0) - \frac{\mathrm{Sin}\,(2\omega_0 t + 2\theta - \alpha\omega_0)}{2\omega_0} + \frac{\mathrm{Sin}\,(\omega_0\alpha+2\theta)}{2\omega_0}\right\}$$

for $t \geqslant \alpha$. (1)

This result shows that the initial transient weight behavior of the ALE under sinusoidal inputs or possibly high signal-to-noise ratio inputs consists of:

a. a ramp shifted in time by α and whose slope is given by $A^2\mu/\Delta\ \mathrm{Cos}\,(\alpha\omega_0)$,

b. a sinusoidal variation of amplitude $A^2\mu/2\omega_0\Delta$, frequency $2\omega_0$, and phase $2\theta-\alpha\omega_0$,

c. a constant term given by $A^2\mu/2\omega_0\Delta\ \mathrm{Sin}\,(\omega_0\alpha+2\theta)$.

TEST CASE

To determine how accurately the result given by Equation 1 approximates the transient weight behavior of the ALE, a digital computer program in FORTRAN V language was written. The following parameters were utilized in the digital simulation of the ALE:

a. $\theta = 0°$

b. $\delta = 0$

c. $A = 1$

d. $T = 1$

e. $\omega_0 = 0.2\pi$

f. $\mu = .01$

g. $\Delta = 1.0$

For this case the theoretical w_ℓ reduces to

$$W_\ell(t) = .01\ [(t - \ell)\ \text{Cos}\ 36°\ell - .8\ \text{Sin}\ (.4\pi t - 36°\ell) + .8\ \text{Sin}\ 36°\ell]\,.$$

Setting $t = nT$

$$W_\ell(n) = .01\ [(n - \ell)\ \text{Cos}\ 36°\ell - .8\ \text{Sin}\ (72°n - 36°\ell) + .8\ \text{Sin}\ 36°\ell]\,. \qquad (3)$$

SIMULATION RESULTS

Figures 2, 3 and 4 show a comparison of the results obtained via a digital simulation of the ALE filter and the theoretical results utilizing Equation 3. It should be kept in mind that Equation 3 was obtained via continuous integration of the cross-correlation vector. This is equivalent to a system whose time increments T are small. For this reason this expression is termed "Continuous Theoretical." For other cases, it might be more meaningful to compare simulated results with the discrete direct solution

$$W_\ell(mT) = \frac{2\mu}{T} \sum_{n=\frac{\delta}{T}+\ell}^{n=m} x(nT)\, x(nT - \delta - \ell T)\,. \qquad (4)$$

The term given by Equation 4 is called "Discrete Theoretical." It can be seen from the results that both representations agreed well with the experimental data. Figure 5 shows the experimental transient and steady state behavior of weights ω_0, ω_1 and ω_2 superimposed on one graph. Here the ramp and oscillatory start up phase is more evident since the time index n has been compressed.

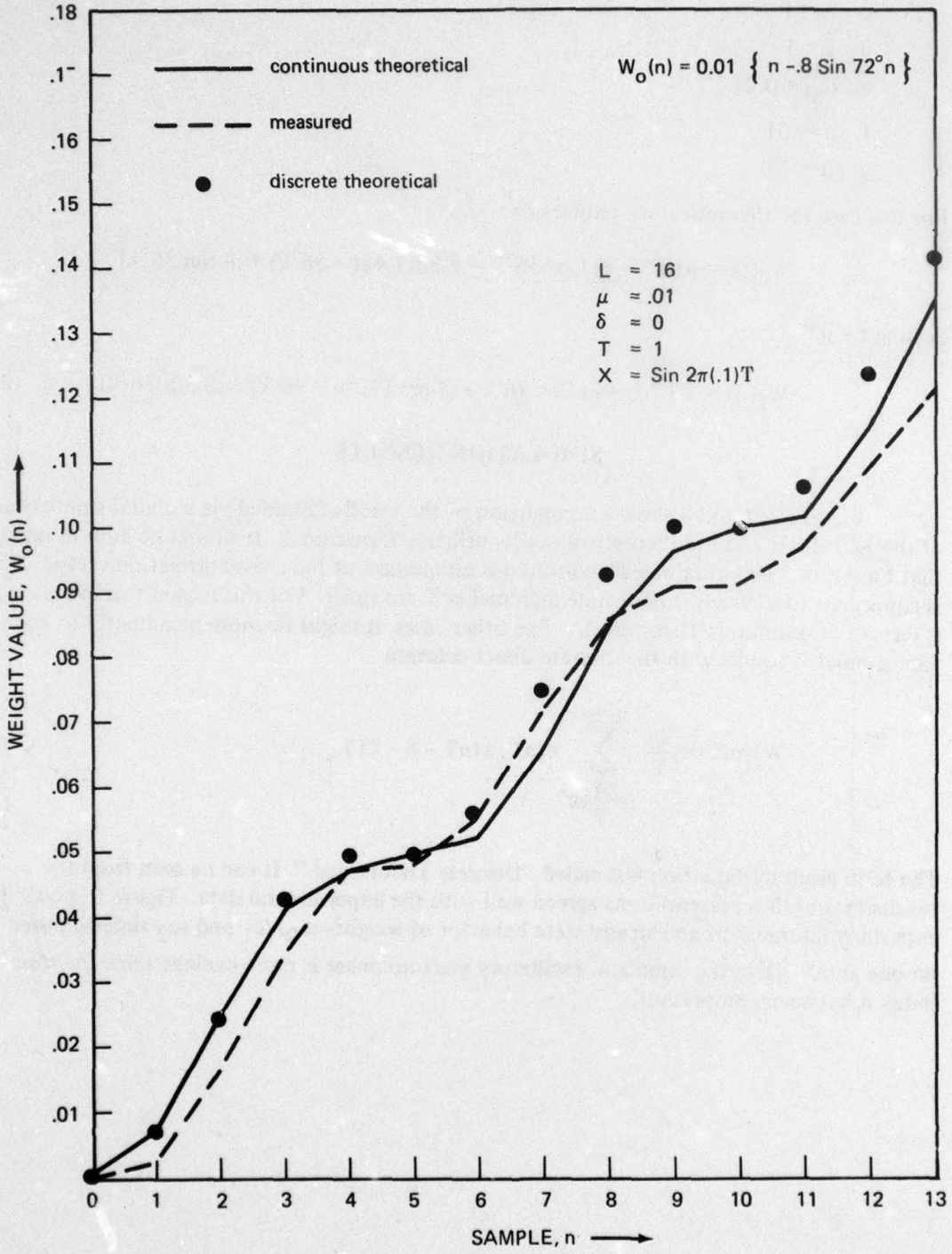

Figure 2. Initial transient response of first weight ($W_o(n)$) of the ALE.

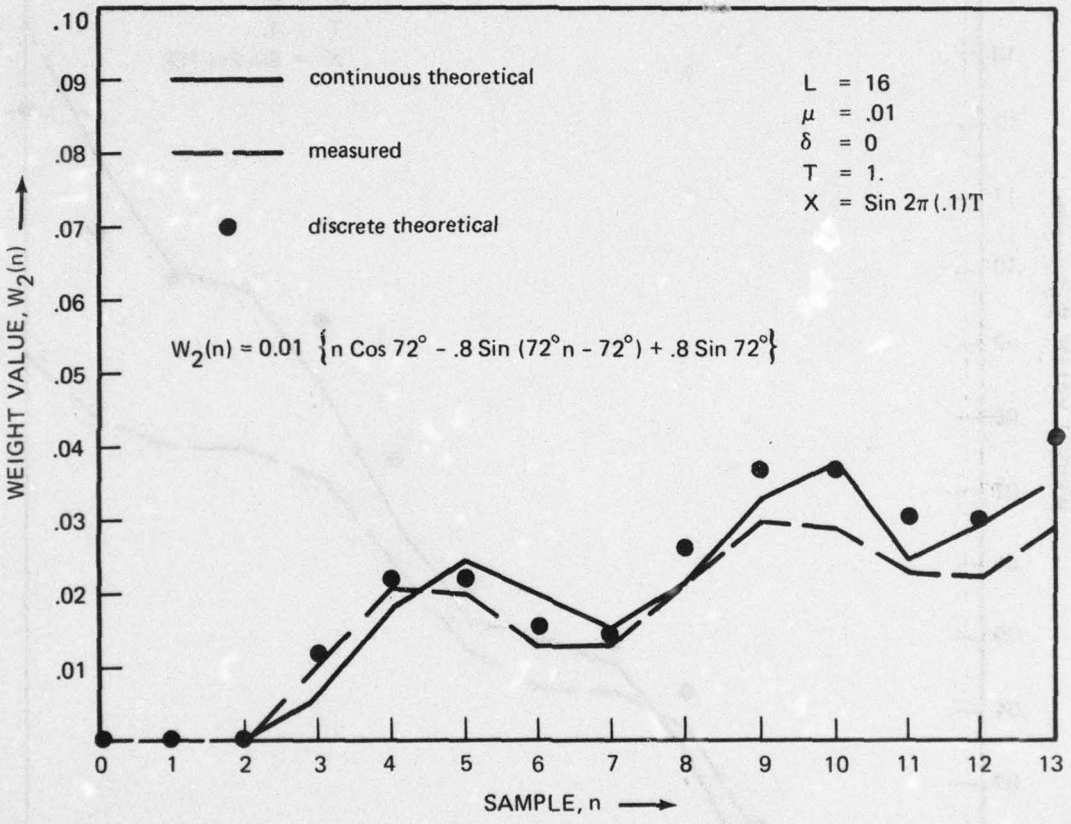

Figure 3. Initial transient response of the third weight ($W_2(n)$) of the ALE.

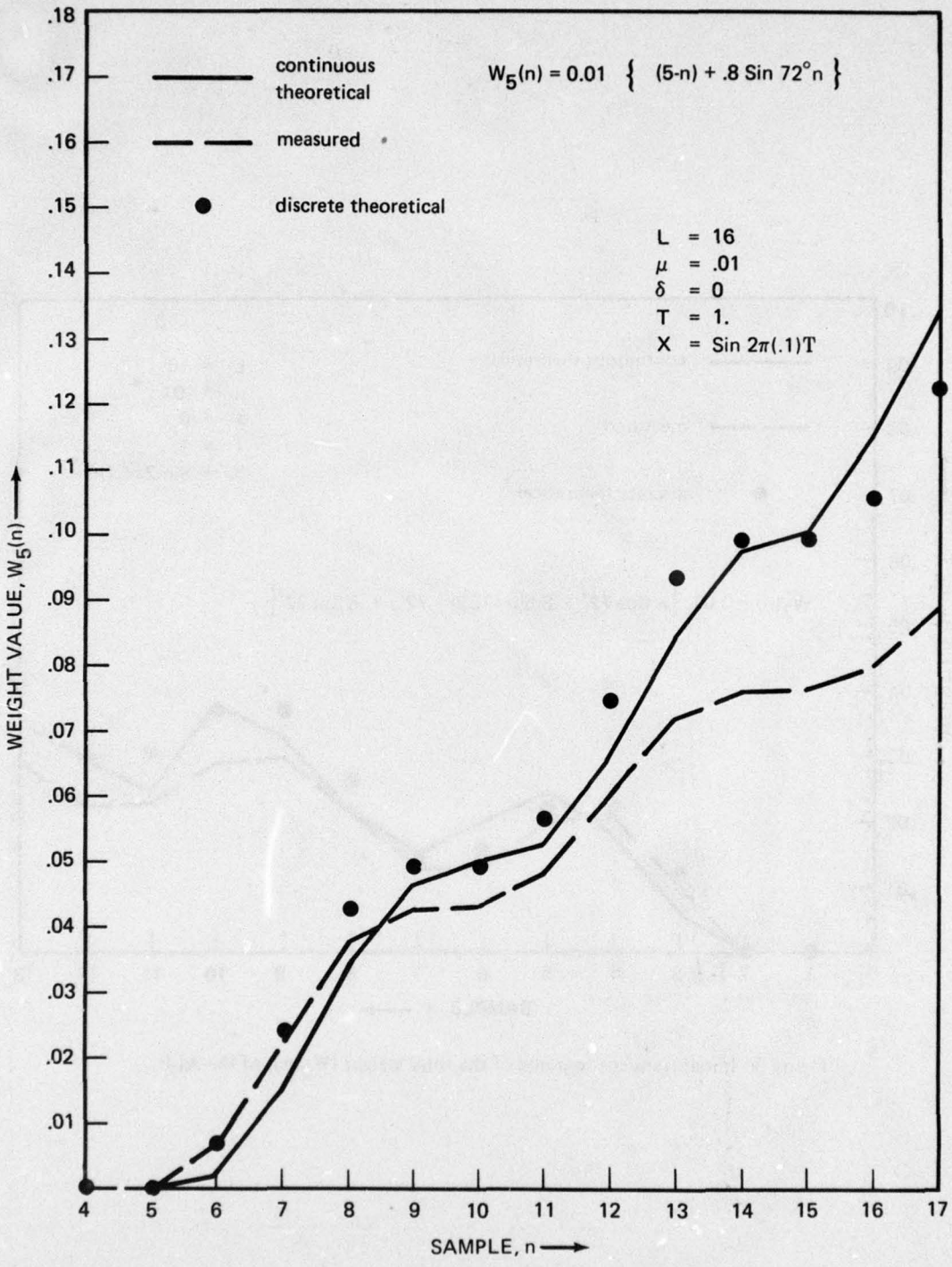

Figure 4. Initial transient response of the sixth weight ($W_5(n)$) of the ALE.

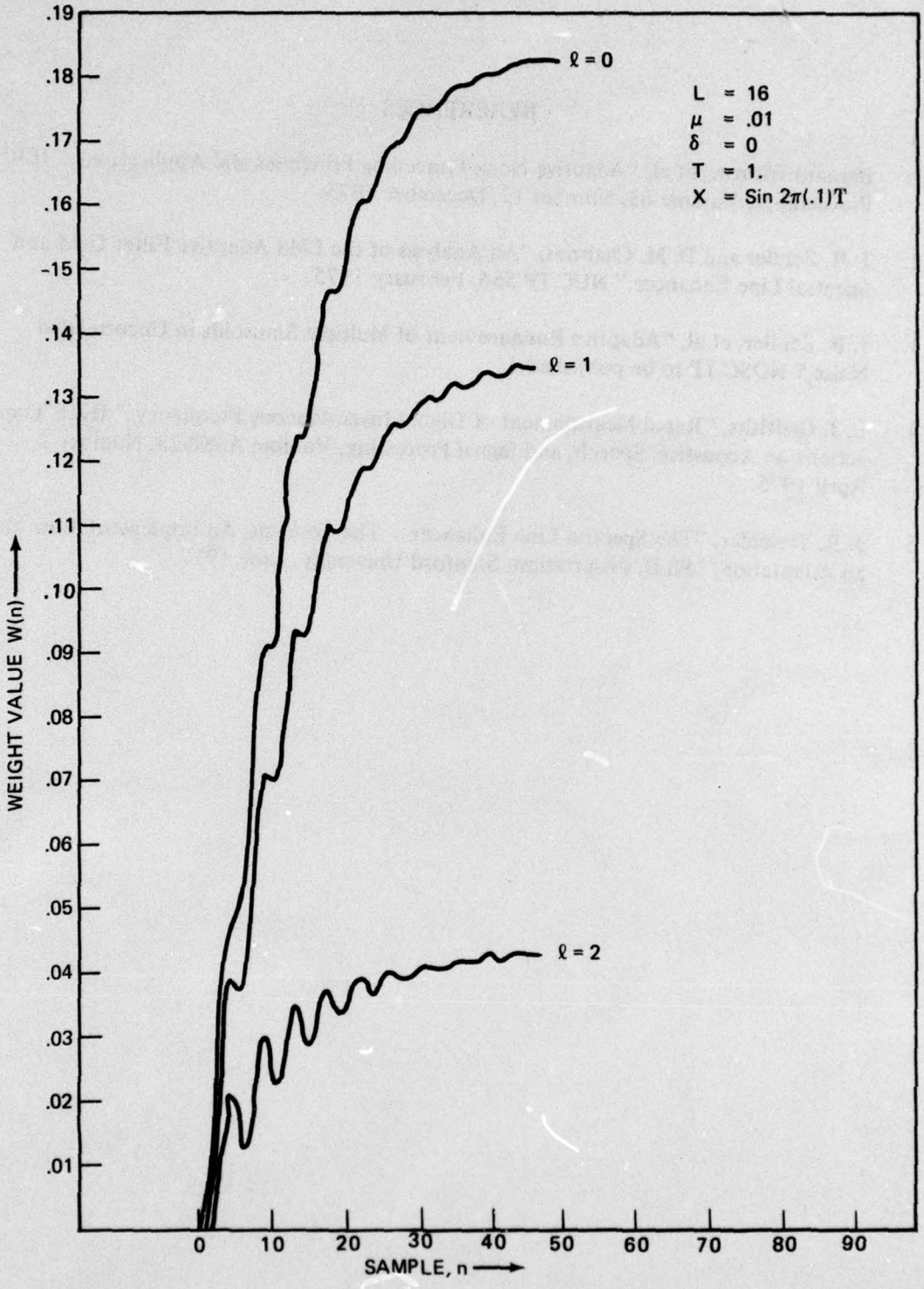

Figure 5. Experimental behavior of W_0, W_1 and W_2 of the ALE.

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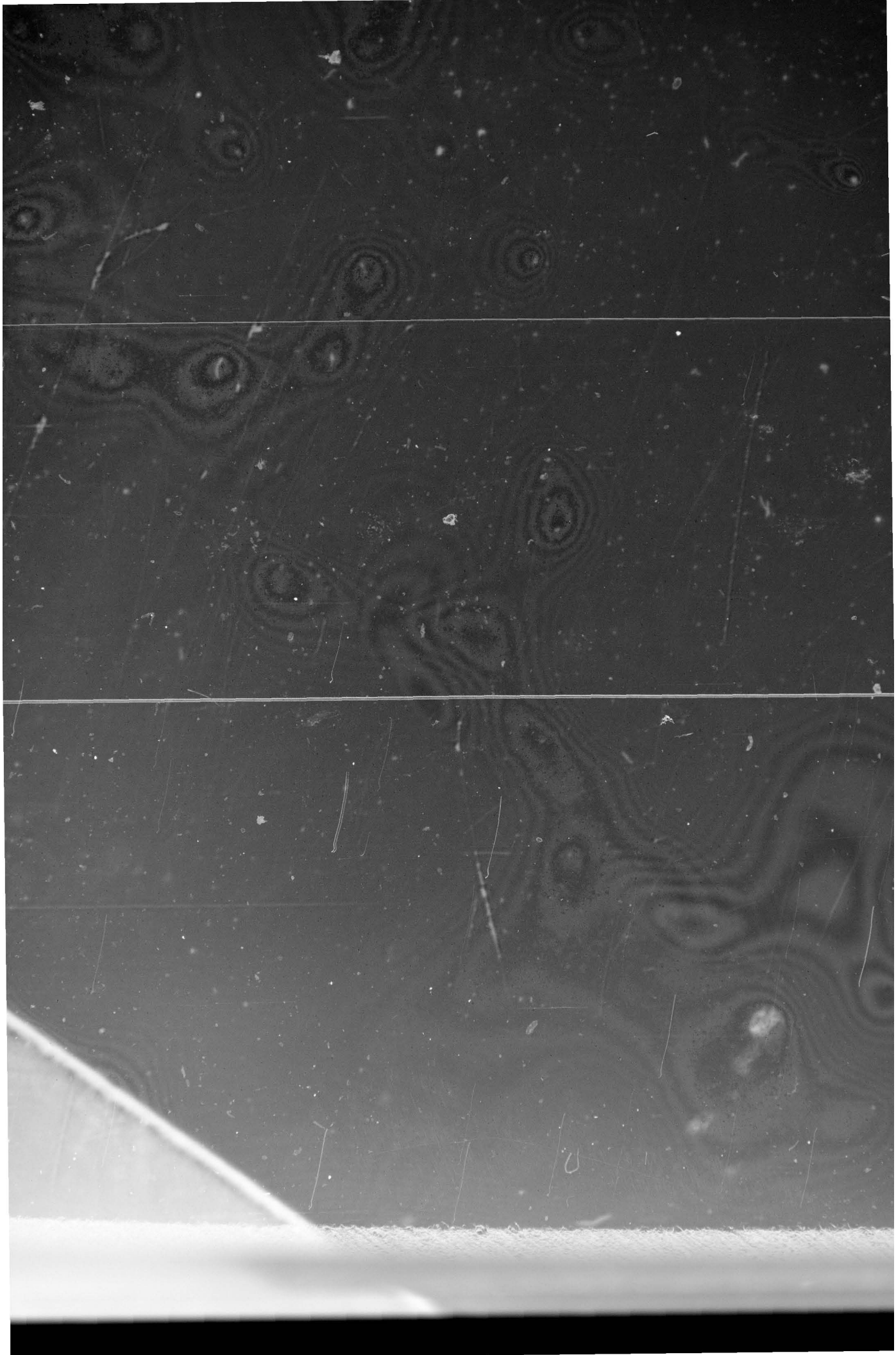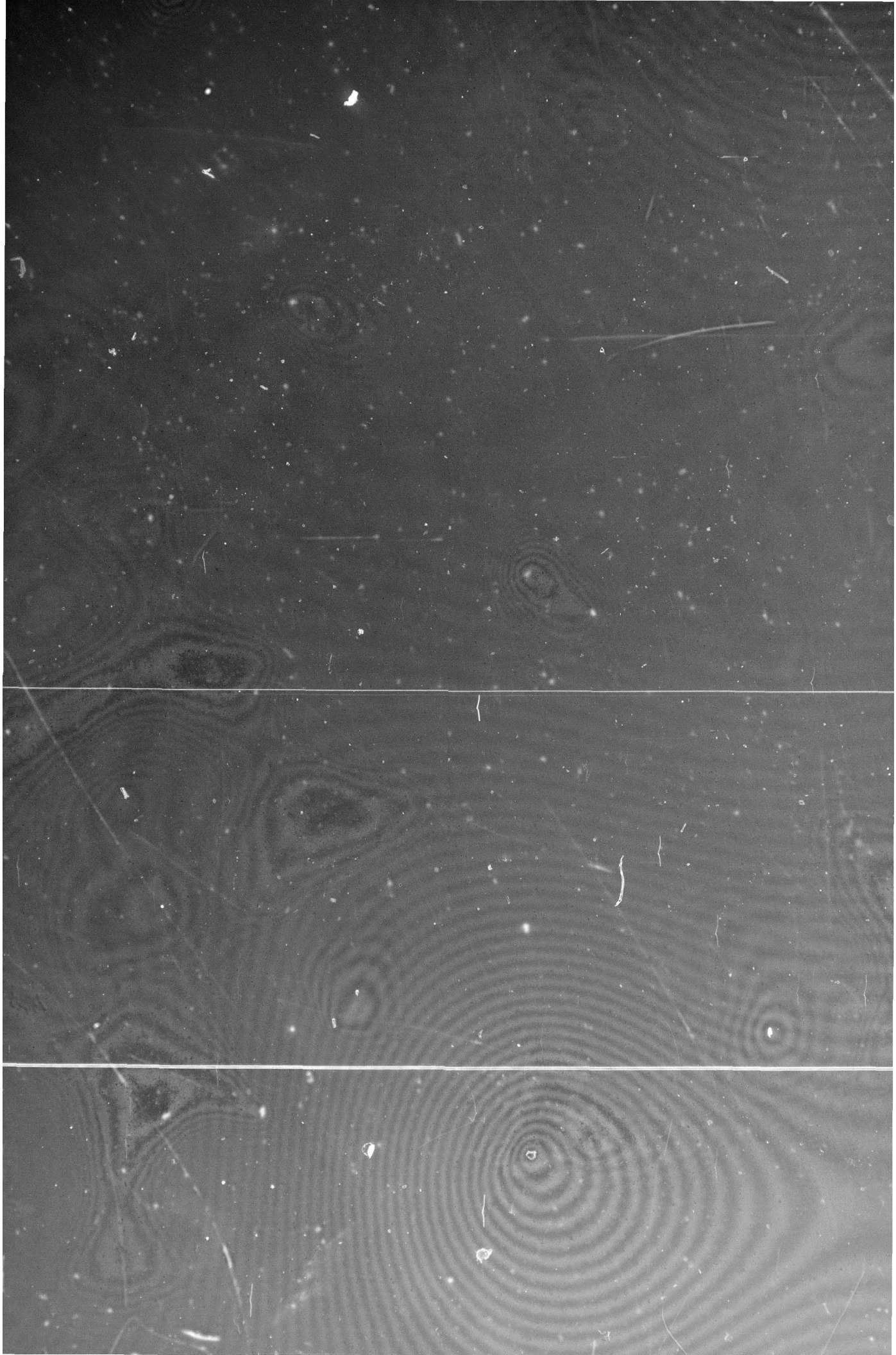
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